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Evolution of low mass stars with the new stellar turbulent convection model

R. Civelek and N. Kızıloğlu

Physics Department, Middle East Technical University, Ankara 06531, Turkey

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Abstract. Canuto and Mazzitelli have proposed an alternative theory to the standard Mixing Length Theory for the treatment of turbulent convection in stars. In this study, we explore the effects of the new treatment of convection on the 0.9, 0.8, 0.7, 0.6 and 0.5 $M_{\odot}$ stars during the pre-main sequence evolution.

Two different cases, with and without an adjustable parameter in the mixing length expression, are applied to the evolution of the Sun starting from the Hayashi track to the present position. We then compare the zero-age-main sequence models of low mass stars obtained with the same input physics, the same code but with different treatments of turbulent convection. We also compare the models with observed properties of some stars in the same mass range.

Key words: stars: evolution - stars: low mass - stars: convection

1. Introduction

In astrophysics, convection has most often been treated within the mixing-length approximation (Vitense 1953; Böhm-Vitense 1958; Cox and Giuli 1968). In the theory, the mixing length, α, is the effective mean-free-path of a turbulent fluid element and it is assumed to be proportional to the local pressure scale height (H_p) : $\lambda = \alpha\ H_p$. The mixing-length theory (MLT) treats the convective turbulence as incompressible and assumes a single eddy model. There are some efforts in the literature to improve the MLT (Gough 1977; Chan et al. 1981; Lester et al. 1982; Canuto 1989; 1990; Lumer 1990). More recently, some new approaches to the problem of stellar turbulent convection have been proposed (Canuto and Mazzitelli 1991, 1992; Grossman et al. 1992; Canuto 1992,1993; Grossman and Narayan 1993).

The motivation of this study is to see the effect of the convection theory of Canuto and Mazzitelli (Canuto and Mazitelli 1991), which is one of the new approaches to turbulent convection in the stars, on low mass stars. In the Canuto and Mazzitelli model (CM model), the full spectrum of eddies is treated and a new formula is proposed for the turbulent convective flux. In addition to the new convective flux expression , the mixing length at any point in the convective region is defined as the distance (z) to the top of the convective zone : λ = z , which is free of the adjustible parameter α. They applied their new turbulence model to the evolution of $1 M_{\odot}$ star and 0.8 $M_{\odot}$ star near the main sequence phase and along the red giant branch by using both $\lambda = \alpha\ H_p$ and λ = z. CM model was also considered for red giant stars of Population I by D'Antona et al. (1992).

In the present study, starting from the Hayashi track where the stars are fully convective, to the zero-age main sequence, evolutions of 0.9 ,0.8 ,0.7 ,0.6 and 0.5 $M_{\odot}$ stars were studied by the use of new expression of convective fluxes of Canuto and Mazzitelli (1991). Theoretical models of low mass stars have been obtained by a number of authors considering a wide range of metallicity and rotation, as well as different treatments of molecular opacity, electron screening factors and other physics (Grossman et al. 1974; Vandenberg et al. 1983; Neece 1984; D'Antona and Mazzitelli 1985; Kızıloğlu 1989, 1991). The particular code of interest here is the modified code of Ezer (Ezer 1972; Eryurt-Ezer et al. 1986). To test the application of CM model to this code, the evolution of the Sun was studied. Section 2 describes the procedure followed in obtaining the models. The comparison of the new models thus obtained with the old ones and with some observed stars in the same mass range are given in Section 3.

2. Procedure

Stars with 1, 0.9, 0.8, 0.7, 0.6 and 0.5 solar masses were constructed using Ezer's evolutionary code modified according to the new convective theory of CM. The evolutionary study was carried out using the Henyey method which was applied from the center all the way to the surface. The method of computations and other physical inputs, except the treatment of convection, have already been discussed in the study of Kızıloğlu and Civelek (1992). The evolutionary sequences of models were started at the threshold of the stability point at which the models become stable against gravitational collapse and were evolved up to zero age main sequence. The solar models were evolved up to the present age of 4.5 10^9 years from the same point.

The turbulent convective flux is given by
F_c = K T H_p^{-1} (∇ - ∇_{ad}) ϕ

Send offprint requests to: R. Civelek

where K is the turbulent conductivity and ∇ and ∇_{ad} are the actual and adiabatic temperature gradients. The ratio of the turbulent thermal conductivity to the radiative conductivity ϕ is a function of (∇ - ∇_{ad}) and A which is a parameter depending on local variables (Canuto and Mazzitelli 1991). In MLT models ϕ is given by

$$\phi = 9/8(4A^2(\nabla - \nabla_{ad}))^{-1}[(1 + 4A^2(\nabla - \nabla_{ad}))^{1/2} - 1]^3$$

Using two modern theories of turbulence, Canuto and Mazzitelli defined ϕ as

$$\phi = a_1(4A^2(\nabla - \nabla_{ad}))^m[(1 + 4a_2A^2(\nabla - \nabla_{ad}))^n - 1]^p$$

The coefficients a_1, a_2, m, n and p are given in the paper of CM. It is also known that $F_{total} = F_{radiative} + F_{convective}$ or

$$\nabla = (\nabla_r + \phi\nabla_{ad})(1 + \phi)^{-1}$$

where ∇_r is radiative temperature gradient. So, in the CM paper, it was shown that for a given A and (∇_r - ∇_{ad}), the (∇ - ∇_{ad}) can be calculated. A matrix of (∇ - ∇_{ad}) values was given in the Table 1 of the same paper. In this study, instead of using Table 1 of CM paper, the convective gradients were found by using a package of soubroutines for overadiabatic convection which was supplied to us by Mazzitelli. In this package, for the non-local treatment of CM (λ = z case), instead of assuming 'z' as the distance from the top of the convection, z was taken to be the harmonic mean between the distance to the top and to the bottom of the convection.

3. Results and Discussion

The MLT and CM models with $\lambda = \alpha\ H_p$ and λ=z were applied to the Sun from the treshold of stability point to the present position. The two models reach the observed luminosity of the Sun after 4.5 10^9 years with initial values of hydrogen X= 0.753 and X= 0.754 respectively. Using the old mixing-length expression, $\lambda =\alpha\ H_p$, the observed position of the Sun in the HR diagram is obtained with α=1.35 for the MLT model and α=0.83 for the CM theory. These results are consistent with those of Canuto and Mazzitelli (1991); to fit the solar data requires $\alpha > 1$ in MLT, while $\alpha < 1$ by the CM model. We could not obtained a good fit to the effective temperature of the Sun with either of the MLT and CM models with no adjustable parameters in the mixing length expression λ = z. MLT with λ = z fails to fit the effective temperature of the Sun by 7 %, while CM theory fails by 6 %. However, Canuto and Mazzitelli found the observed effective temperature within 0.2 % by using CM theory and within 2 % by using MLT without the adjustable parameter. The reason for the discrepancy might be the difference in starting points. In the study of Canuto and Mazzitelli (1991), the starting point to the evolution is homogeneous main sequence where hydrogen is thought to be homogeneous and He^3 to be in equilibrium. On the other hand, we started the evolution from the threshold of the stability point; from there on the star contracts toward the main sequence as a fully convective star up to the age of 2x10^6 yr. During the contraction, the accumulation of He^3 inside the Sun was also followed. As already noted, in the present comparative study, the old and new theories were each applied throughout the whole evolution. We further explored whether the discrepancy between the results of Canuto and Mazzitelli (1991) and ours was really due to the difference in starting points. For this purpose, we computed one more evolutionary track for one solar mass star from the homogeneous main sequence to the present position by applying the CM model without the adjustable parameter. The result was a solar model with the same luminosity as the present sun but an effective temperature within 7 %. Consequently, we estimate that neither the MLT nor the CM model, without the adjustable parameter, fits the effective temperature of the sun, whatever the starting point is.

In order to find the zero-age-main sequence (ZAMS) of low mass stars, we use the CM theory throughout the evolution in the treatment of the convection, setting λ = 0.83 H_p. For comparison the ZAMS positions of the stars were also obtained for λ = z following the same procedure. The physical properties of stellar models in the mass range 0.9 and 0.5 $M_\odot$ which were obtained using CM formalisms with λ = 0.83 H_p and λ = z are summarized in Table 1 together with MLT models for λ = 1.35 H_p. The first column shows the masses of stars in units of solar mass. The ZAMS time, luminosity, radius, fraction of mass in the convective envelope are presented in the columns 2 to 5, respectively.

The ZAMS positions of the low mass stars in the considered mass range with old and new mixing length theory for two different length expressions were plotted in the Hertzsprung-Russell diagram in the Figure 1. As seen from the figure, the ZAMS positions of low mass stars do not change by using two different convection theories as far as λ is taken as a multiple of H_p. This is what Canuto and Mazzitelli concluded about the Main Sequence structures of 1 and 0.8 $M_\odot$ stars (Canuto and Mazzitelli 1991, 1992). On the other hand, the use of the smaller mixing length, that is, λ = z, is more effective in determining the structure of a star since a considerable departure from the original MLT and CM theory with $\lambda = \alpha H_p$ was seen especially for masses greater than 0.7 $M_\odot$. Two different formalisms give almost the same luminosities but larger radii in the case of λ = z, which yields lower surface temperatures. Using CM theory with λ =z enlarges the radius of a 0.9 $M_\odot$ star by about 11 % while it enlarges that of a 0.5 $M_\odot$ star by about 2 % at the ZAMS. It seems from the H-R diagram that for 0.9 and 0.8 $M_\odot$ stars the convection theory is important in determining the stellar structure parameters, but for the other low mass stars whose surface temperatures are lower, the surface conditions are important and the luminosities are determined entirely by the photospheric conditions. In Figure 1, some of the observed low mass stars taken from Popper (1983), Harmanec (1988) and Vandenberg et al. (1983)were also plotted. Popper's data includes only visual binary systems with parallaxes greater than $0''.09$. Harmanec gave the mean and extreme observed masses, radii and bolometric magnitudes of normal Main Sequence stars as a function of spectral type. The data which is tabulated by Vandenberg et al. is a compilation of M_{bol} and T_{eff} for late type dwarfs. The figure shows that the MLT and the CM models with lambda depending on H_p are more succesfull in representing the observed data of low mass stars, although the test is not discriminant enough. The agreement between the theoretical and observed values is still unsatisfactory.

The main aim of this paper was to study the pre-main sequence evolution, where the stars are mostly convective, of the low mass stars with the new formalism given by Canuto and Mazzitelli (1991). The use of the CM model itself does not differ so much from MLT unless a smaller mixing length, that is λ = z rather than $\lambda = \alpha\ H_p$, is used. The results indicate that the new formalism without an adjustable parameter gives lower surface temperatures than the MLT one. We see that a change in the value of α or in the convective theory can cause

Table 1. Some physical properties of ZAMS models with two different theory of convections

Fig. 1. The ZAMS positions of stars of indicated masses in the HR diagram. Triangle, square and dot signs indicate the observed data of Popper (1983), Harmanec (1988) and Vandenberg et al. (1983).

a significant effect on the overall structure of the star since nearly all the physical parameters are sensitive to the type of mixing length expressions used.

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