DYNAMICS OF THE NEUTRON STAR INTERIOR

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1. Introduction

This lecture summarizes the basic ideas of the dynamics of rotating superfluids in neutron stars (Section 2), and discusses the application to pulsar glitch models (Section 3). Energy dissipation in the neutron star interior is discussed in Section 4. Excellent general reviews of superfluidity and superconductivity, in connection with the neutron star context can be found in the contributions of Sauls to this volume and to the earlier NATO ASI on "Timing Neutron Stars" (Sauls 1989).

2. The Rotating Superfluid

The equilibrium state of a system with constant angular momentum $\mathbf{J}$ is obtained by minimizing the free energy $E' = E - \mathbf{\Omega} \cdot \mathbf{J}$. The free energy is minimum in a state of rigid body rotation

$$\mathbf{v}(\mathbf{r}) = \mathbf{\Omega} \times \mathbf{r} \tag{1}$$

$$\nabla \times \mathbf{v} = 2\mathbf{\Omega} \tag{2}$$

Normal (ie non-superfluid) matter in differential rotation will evolve towards the equilibrium state of rigid body rotation as viscous processes transport angular momentum and even out differential rotation.

A superfluid has a very different way of reaching the free energy minimum: it rotates by having an array of quantized vortices. In a superfluid a macroscopic number of particles all occupy the same quantum mechanical state. The macroscopic velocity $\mathbf{v}_s$ of the fluid is given by the gradient of the quantum mechanical wavefunction of this ground state:

$$\mathbf{v}_s = \frac{\hbar}{2m_n}\nabla\phi \tag{3}$$

where ϕ is the phase of the wavefunction. (The "particle" mass is taken to be twice the neutron mass m_n since the neutron star superfluid is made up of Cooper pairs of neutrons.) Being the gradient of a scalar (potential flow), this velocity cannot support rotation, ie $\nabla \times \mathbf{v}_s$ is zero, unless there are singularities in ϕ. In polar coordinates (r, θ) in the plane perpendicular to a line singularity, with the singularity as origin, the phase ϕ should depend only on θ if the singularity is to contribute to $\nabla \times \mathbf{v}_s$ but not to $\nabla \cdot \mathbf{v}_s$. For a single valued wavefunction the phase satisfies

$$\phi(\theta + 2\pi) = \phi(\theta) \tag{4}$$

$$\phi(\theta) = n\theta \tag{5}$$

where n is an integer. The superfluid velocity circulating the singularity is obtained from Eqs.(3) and (5):

$$\mathbf{v}_s(\mathbf{r}) = \frac{n\kappa}{2\pi r}\hat{\theta} \tag{6}$$

where θ is the unit vector in the azimuthal direction around the singularity and $\kappa = h/2m_n$ is the quantum of vorticity for the neutron superfluid, h being the Planck constant. Our singularity is a quantized vortex line carrying circulation (vorticity):

$$\oint \mathbf{v}_s \cdot \mathbf{dl} = n\kappa \tag{7}$$

$$\nabla \times \mathbf{v}_s = n\kappa\delta(\mathbf{r}) \tag{8}$$

The value of the vorticity quantum κ is $2 \times 10^{-3} cm^2 s^{-1}$. The quantized vortex line is not a physical singularity: each vortex line has a core, a cylinder made up of normal matter. The local velocity field around the vortex line diverges as r^{-1} and the kinetic energy per particle in the superfluid phase diverges as r^{-2}. At a radius ξ called the superfluid coherence length, the kinetic energy per particle in the superfluid phase exceeds the condensation energy, the energy difference per particle between the normal and superfluid phase. The vortex core radius ξ is typically of the order of 10-50 fermis in the neutron star superfluid.

The idea that superfluid rotation is set up by quantized vortex lines was first put forward by Onsager and independently by Feynman, and is described in a very lucid manner in Feynman's book on Statistical Physics (Feynman 1981). A superfluid can reach the minimum free energy state by setting up a uniform density array of lines each carrying a single vortex quantum (n=1). At the rotation rates typical of neutron stars the energy cost of introducing the vortex lines is negligible compared to the gain in rotational free energy.

At a macroscopic distance R from the rotation axis the collective effect of the array of vortex lines is to set up a macroscopic superfluid velocity field which is very close to rigid body rotation. This can be seen by considering the circulation along a contour at R from the rotation axis. By cylindrical symmetry, $\mathbf{v}_s(R)$ will be in the azimuthal direction,

$$\oint_R \mathbf{v}_s \cdot \mathbf{dl} = 2\pi R^2 \Omega(R) \tag{9}$$

while the Stokes theorem relates this to the discrete contributions of the quantized vortices:

$$\oint_R \mathbf{v}_s \cdot \mathbf{dl} = \int \nabla \times \mathbf{v}_s \cdot \mathbf{dS} = \int \kappa \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \cdot \mathbf{dS} = N(R)\kappa = \int_0^R \kappa n(r) 2\pi r dr. \tag{10}$$

where N(R) is the total number of vortices within the contour and n(r) is the number of vortices per unit area. Differentiating Eqs.(9) and (10), we find that the equilibrium state of rigid body rotation at rate Ω is obtained with a uniform vortex density n_0 while a state of differential rotation $\Omega(r)$ is set up by a corresponding vortex density distribution n(r):

$$n_0 \kappa = 2\Omega \tag{11}$$

$$n(r)\kappa = 2\Omega(r) + r\frac{\partial \Omega}{\partial r}. \tag{12}$$

For a pulsar rotating at $\Omega = 100$ rad/s^{-1}, the vortex density n_0 is 10^5 cm^{-2}, the average vortex spacing $\ell \approx 3 \times 10^{-3}$ cm, and the total number of vortex lines in the neutron star is of the order of 10^{17}. In the absence of forces on vortex lines the lines also move with the local superfluid, so that the array of vortices is in rigid body rotation, together with the superfluid whose rotation the vortex lines collectively set up.

The spindown (or spinup) of a rotating fluid requires a change in the vortex density. For spindown at distance R from the rotation axis, vortices must move through the contour at R in the radially outward direction from the rotation axis. The spindown rate of a superfluid is proportional to the radially outward *vortex current*,

$$\dot{\Omega}_s(r) = -\frac{\kappa n(r) V_r}{r} \tag{13}$$

where V_r is the average radially outward velocity of the vortices. V_r is a function of the lag $\omega = \Omega_s - \Omega_c$ between the rotation rate Ω_s of the superfluid, and the rotation rate Ω_c of the neutron star's outer crust and normal matter coupled to it. The outer crust, which is directly coupled to the pulsar magnetospheric torques, leads the spindown, so that $\Omega_s > \Omega_c$. The form of the function V_r depends on the physics of the interaction between normal matter and vortex lines, and the dynamical response of the vortex lines to this interaction.

The equation of motion for a quantized vortex line is the same as for a vortex in a classical fluid. As the Kelvin circulation theorem states, in the absence of forces a vortex will move with the fluid. When a force is applied on a vortex line, it moves at a relative velocity with respect to the fluid, in a direction perpendicular to both the force and the vortex line. For a quantized vortex line the "Magnus" equation of motion is

$$\mathbf{f} = \rho\kappa \times (\mathbf{v}_s - \mathbf{v}_L) \tag{14}$$

where $\mathbf{v}_s$ and $\mathbf{v}_L$ denote the superfluid and vortex line velocities respectively and ρ is the density of the superfluid. This equation of motion for the line derives from the Newtonian mechanics of the fluid: If the line moves at relative velocity $\delta\mathbf{v} = \mathbf{v}_L - \mathbf{v}_s$, the local velocity field $\kappa\theta/2\pi r$ circulating around the line will superpose with $\delta\mathbf{v}$ to give higher kinetic energies on the side of the line where $\delta\mathbf{v}\cdot\theta > 0$, and lower on the other side. This situation leads to a kinetic energy gradient (ie a gradient of the Bernoulli pressure, a hydrodynamic force) in the fluid, in a direction transverse to $\delta\mathbf{v}$; to sustain such a motion of the line, a physical force (per unit length) $\mathbf{f}$ must be present to cancel the transverse hydrodynamic force. This hydrodynamic force, $\rho\kappa\times(\mathbf{v}_s-\mathbf{v}_L)$ is usually called the Magnus force. Like the centrifugal force, it is misleading to think of this as a particular sort of physical force; rather, the right hand side of Eq.(14) describes the motion of a vortex line under a physical force $\mathbf{f}$ appearing on the left side. This equation of motion applies to the two qualitatively different situations facing vortex lines in the neutron star: drag forces in the neutron star core and pinning forces in the inner crust superfluid.

The crust of a neutron star is expected to be solid, made of crystals of various kinds. The "core" of a neutron star, where there is no crystal lattice, starts at densities of about $\sim 2\times 10^{14}$gm cm^{-3} and contains almost all of the mass, volume and moment of inertia of the star. In this core region, vortex lines are in a homogeneous medium composed of the neutron superfluid, superconducting protons and electrons in the normal state. The electrons spin down with the neutron star crust to which they are coupled by normal matter viscous processes. As the superfluid and the vortex lines lag the electrons in spin down, there is a relative motion, in the azimuthal direction with respect to the star's rotation axis, between the vortex lines and the electrons. As the electrons interact with the normal matter cores of the vortex lines by various dissipative mechanisms, there are drag forces on the vortex lines, acting in the azimuthal direction. In response, according to

Eq.(14) the vortex lines move radially outward and the superfluid thereby spins down, following the normal matter at some lag.

The crust of a neutron star is expected to be solid, made of lattices of nuclei (Pethick and Ravenhall 1991, 1995). Apart from the very large densities and pressures in the neutron star, these lattices are also characterized by the presence of excess neutrons, in continuum Bloch states of the lattice, at densities above about $\sim 4 \times 10^{11}$gm cm^{-3}, which is called the neutron drip density. These continuum neutrons are superfluid, so that in the "inner crust", above the neutron drip density, there is a superfluid in coexistence with the lattice. The nuclei forming the lattice are pinning sites for the vortex lines if the energy cost for a vortex line core is lowered if it happens to pass through a nucleus. The nuclei (the sites of bound protons and neutrons) may be cylindrical rods or planar slabs at the highest densities of the inner crust (Pethick and Ravenhall 1995). Independently of the nature and strength of the pinning, the Magnus equation of motion implies that pinning forces are in the radially inward direction. As long as vortex lines remain pinned, the superfluid's rotation rate does not change. The lines must move with the lattice to which they are pinned. Since the lattice is spinning down under the accretion torque, as part of the normal matter crust of the star, the pinned lines have to be moving with respect to the superfluid, the relative velocity $\mathbf{v}_L - \mathbf{v}_s$ being in the $-\phi$ direction. To enforce this motion the pinning forces supplied by the lattice are in the $-\mathbf{r}$ direction (towards the rotation axis) according to Eq.(14).

3. Modes of Spindown for the Neutron Star Superfluid

The fractional changes in the spindown rate are of the order of $\Delta\dot{\Omega}/\dot{\Omega} \sim 10^{-3} - 10^{-2}$ in all pulsar glitches and postglitch relaxation. The standard interpretation of this regularity is that the crust superfluid which comprises a few percent of the total moment of inertia is responsible for the postglitch relaxation and the glitches themselves (Alpar and Pines 1989, Alpar 1991, Pines 1991, Alpar 1995). The glitches and postglitch relaxation are then modelled in terms of the angular momentum exchange between the crust superfluid and the rest of the star, including the core superfluid, which contains most of the moment of inertia of the star. The core superfluid must be already coupled to the outer crust and other normal matter on timescales τ short compared to the rise time of the glitch, which is less than two minutes in the Vela pulsar. Such tight coupling between the neutron vortex lines in the homogeneous core superfluid and the normal matter electrons arises from the spontaneous magnetization of the vortex lines by superfluid protons dragged along with superfluid neutrons into circulation around the vortex lines (Alpar, Langer and Sauls 1985, Alpar and Sauls

1988, Sauls 1989). The core superfluid then spins down following the normal matter crust at a tiny lag $\omega = |\dot{\Omega}|\tau$, and is effectively a part of the "normal" matter crust on timescales longer than τ. In the rest of this lecture the "observed crust" or "normal matter crust" will mean the effective crust including the core superfluid. This "crust" contains almost the entire moment of inertia of the star. We shall distinguish from this "the pinned crust superfluid" (or "crust superfluid") which is responsible for the interesting glitch and postglitch dynamics. (There is one major open problem regarding the dynamics of the core superfluid, namely the interaction of the neutron vortex lines in the core with the separate array of quantized magnetic flux lines of the superconducting protons. This problem may be relevant to the glitch dynamics (Sauls 1989) as well as a possible correlation between spindown and magnetic field decay on evolutionary timescales of the neutron star (Srinivasan et al 1990, Bhattacharya and Srinivasan 1991, Bhattacharya 1995)).

With this framework we now turn to a discussion of the modes of spindown of the crust superfluid. Postglitch spindown typically involves several distinct pieces, characterized by different moments of inertia, relaxation times and modes of relaxation. These distinct pieces make up a total of only a few percent of the moment of inertia. It is therefore a good approximation to model the contribution of each element of the crust superfluid to the observed relaxation of the neutron star separately, in a two component model with the particular part of the crust superfluid as one component. The other component is the "effective" normal matter crust, which is practically the entire star rotating rigidly with the outer crust whose dynamics we observe through pulsar timing. Each such contribution from some part of the crust superfluid is associated with one distinct piece of the observed glitch and postglitch relaxation. The observed relaxation consists of a superposition of the solutions for the normal matter "effective" crust from each of these two component models. Parts of the pinned crustal superfluid having different modes of spindown contribute qualitatively different signatures to the spindown of the observed crust. The different possible signatures are illustrated in the figures.

Figure 1 shows the spindown of a strictly pinned "vortex capacitor" part of the crust superfluid. Spindown occurs only in sudden vortex discharges whenever the lag reaches a critical value ω_{cr} (the glitches). In between glitches the superfluid rotation rate sustained by the pinned vortices remains constant. Corresponding to this, between glitches the observed crust spins down under the external torque alone since there is no continuous spinup internal torque on the observed crust when there is no vortex current. The spindown rate of the observed crust between glitches is $\dot{\Omega}_c = N_{ext}/I_c$. At each glitch, the pinned superfluid and the crust spin down

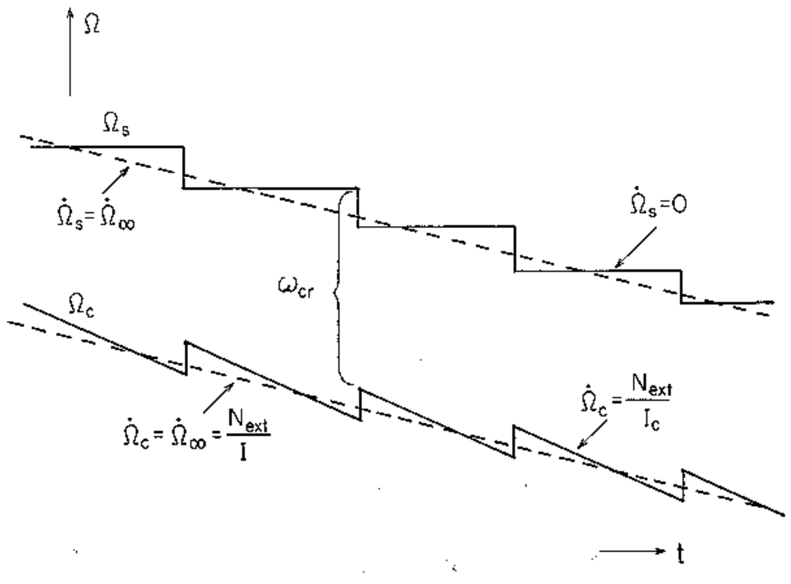

Figure 1.

by $\delta\Omega_s$. Angular momentum conservation relates $\delta\Omega_s$ to the observed glitch in the crust's rotation rate $\Delta\Omega_c$:

$$I_c \Delta\Omega_c = I_s \delta\Omega_s \tag{15}$$

Dynamical equilibrium requires that the pinned superfluid and the crust spin down at the same average rate $\dot{\Omega}_\infty = N_{ext}/(I_c + I_s)$, sharing the external torque in proportion to their respective moments of inertia and keeping the lag constant. The pinned superfluid achieves this only in discrete steps $\delta\Omega_s$ such that

$$(\sum \delta\Omega_s)/(\sum t_g) = |\dot{\Omega}|_\infty \tag{16}$$

where t_g is the time between two successive glitches. Taking into account the spinup steps observed as glitches, and equations (15) and (16), the mean spindown rate of the crust is $\dot{\Omega}_\infty = N_{ext}/(I_c + I_s)$. For the Vela pulsar glitches, Eq.(16) is indeed satisfied. After the postglitch relaxation components are over, there remains a nonrelaxing component of the discrete spinup in each glitch. This means that the vortex capacitor pinned superfluid manages to spin down with glitches alone. The corresponding spinup in the crust is therefore also composed of discrete steps which do not relax, as shown in Fig.1.

Other parts of the pinned crust superfluid which are not completely pinned but allow a continuous vortex current and spindown, for exam-

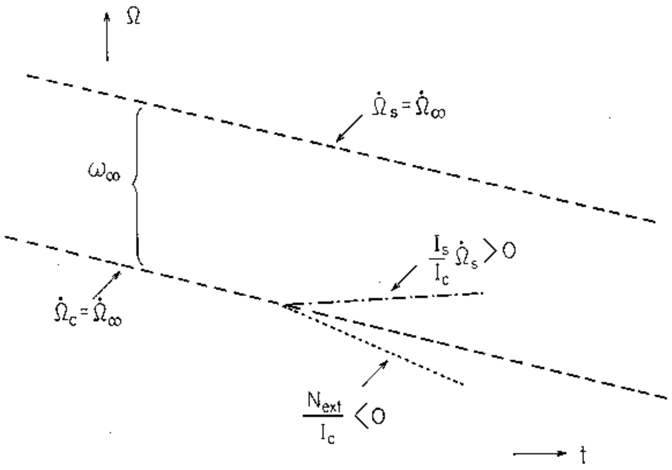

Figure 2.

ple by thermal vortex creep, can reach a dynamical steady state, with the steady state lag $\omega = \Omega_s - \Omega_c \equiv \omega_\infty$ such that the pinned superfluid and the observed crust spin down at the same rate $\dot{\Omega}_\infty$. Fig.2 shows this steady state spindown schematically . The spindown of the superfluid contributes an internal torque on the crust corresponding to a spinup at the rate $-(I_s/I_c)\dot{\Omega}_s > 0$ (dashed-dotted line). Without the coupling of the pinned crust superfluid and the resulting spinup torque, the crust would have spun down under the external pulsar torque alone, at the rate $N_{ext}/I_c < 0$ (dotted line). The resulting spindown of the crust is $\dot{\Omega}_c = N_{ext}/I_c - (I_s/I_c)\dot{\Omega}_s < 0$. In steady state $\dot{\Omega}_c = \dot{\Omega}_s = N_{ext}/I \equiv \dot{\Omega}_\infty$ where $I = I_c + I_s$ is the total moment of inertia (dashed line).

Deviations from the steady state are introduced by the glitches, since Ω_c is observed to increase suddenly, and, in those parts of the pinned crust superfluid through which the suddenly unpinned vortices move at the glitch, Ω_s also changes, undergoing a decrease by $\delta\Omega_s$. Since the spindown of the superfluid, that is, the vortex current, is a function of the lag $\omega = \Omega_s - \Omega_c$, and the spindown of the observed crust is effected by the spinup resulting from the spindown of the pinned superfluid, the glitch related offset of the lag $\omega = \Omega_s - \Omega_c$ will lead to an immediate offset in the spindown rate of both components. As the spindown has to continue, due to the external pulsar torque, the lag will promptly start to evolve back towards its steady

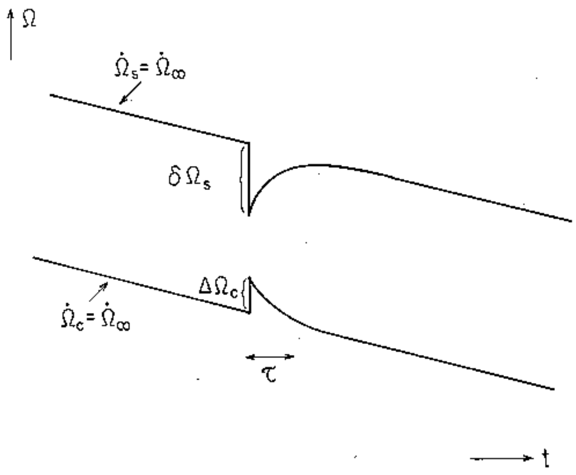

Figure 3.

state value, and hence the spindown rates will start to relax to the steady state configuration. The nature of this postglitch recovery depends on the coupling between the crust and the pinned superfluid. Three possibilities are illustrated in Figures 3-6.

If the radial vortex current has only a linear dependence on $\omega = \Omega_s - \Omega_c$ then the equation of motion for Ω_s is linear, and the response to perturbations in Ω_s and Ω_c is also linear: The change in the spindown rates $\Delta\dot{\Omega}_c$ at the glitch is linear in the perturbations $\Delta\Omega_c$ and $\delta\Omega_s$, and the subsequent relaxation is a simple exponential function of time which is illustrated in Fig.3.

Alternatively the vortex current can have a very nonlinear dependence on ω (Alpar, Cheng and Pines 1989). This is usually the case in vortex creep where the nonlinearity arises from Boltzmann factors whose exponents contain the lag ω which enters through the pinning (activation) energy. As shown in Fig.4, the perturbation away from steady state effectively stops the vortex current and spindown of the superfluid which are very sensitive functions of ω. The crust continues to spindown at the rate N_{ext}/I_c and eventually the steady state lag is reached again after a waiting time $t_g \equiv \delta\Omega_s/|\dot{\Omega}|_c$. Fig.4 shows the corresponding evolution of the spindown rates. (Note that $\delta\omega = \delta\Omega_s + \Delta\Omega_c \cong \delta\Omega_s$ in a glitch with $\Omega_s \to \Omega_s - \delta\Omega_s$ and $\Omega_c \to \Omega_c + \Delta\Omega_c$. By angular momentum conservation

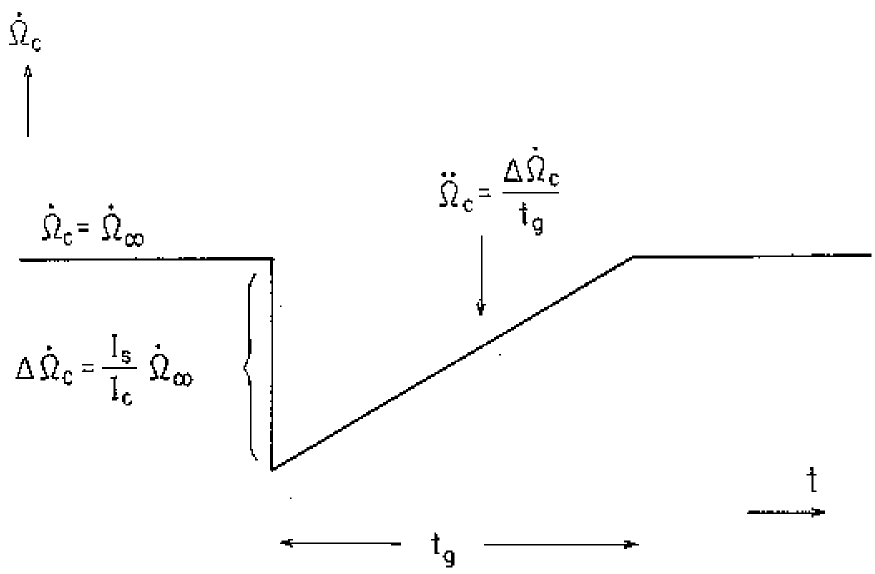

Figure 4.

$I_s\delta\Omega_s = (I_c/I_s)\Delta\Omega_c >> \Delta\Omega_c$ since the effective crust includes the neutron star's core, and has therefore much larger moment of inertia than any component of the pinned crustal superfluid.)

Fig.5 is the time derivative of Fig.4, showing the evolution of the spindown rates corresponding to the nonlinear response shown in Fig.4. The actual nonlinear response observed in the Vela pulsar is not like the model behaviour depicted in Fig.4 and Fig.5. After exponentially relaxing linear response components are over, the remaining spindown rate of the crust undergoes a recovery which is proportional to the time elapsed since the glitch, a constant second derivative $\ddot{\Omega}_c$, as shown in Fig.6. This is simply interpreted as a stacking of the behaviour shown in Fig.5: a distribution of small subregions of the pinned crustal superfluid, each sustaining vortex creep in steady state , are offset by different $\delta\Omega_{s,i}$ at the glitch and each such region, with moment of inertia I_i restarts steady state creep after its own waiting time $t_{g,i} = \delta\Omega_{s,i}/|\dot{\Omega}|_c$. This composite nonlinear response is over when the longest waiting time t_g is reached. The observed behaviour

$$\Delta\dot{\Omega}_c(t) = \Delta\dot{\Omega}_c(0)(1 - t/t_g) \qquad (17)$$

arises if the glitch had involved unpinning of vortices at a uniform density. This is a valid first approximation to the distribution of vortices unpinned at the glitch.

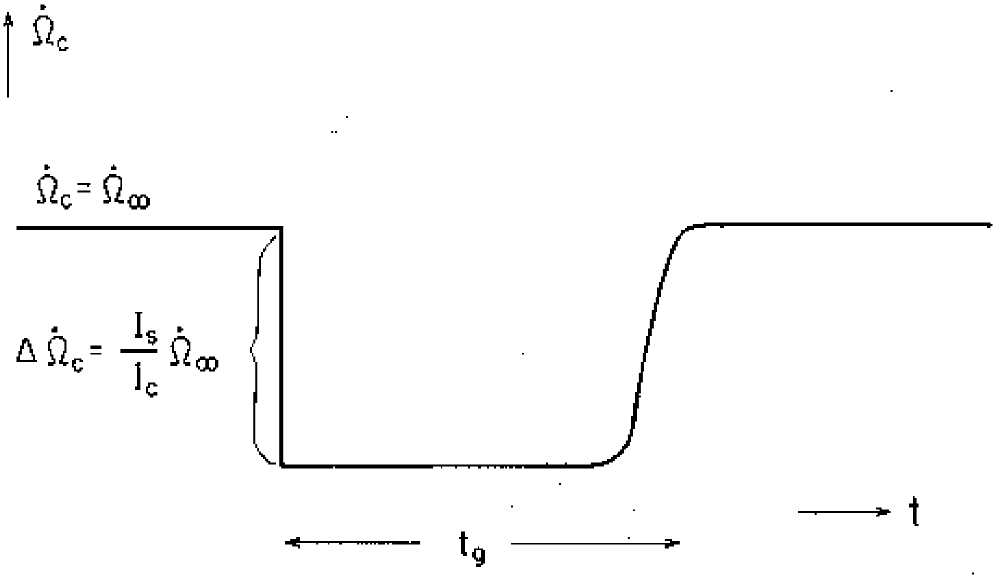

Figure 5.

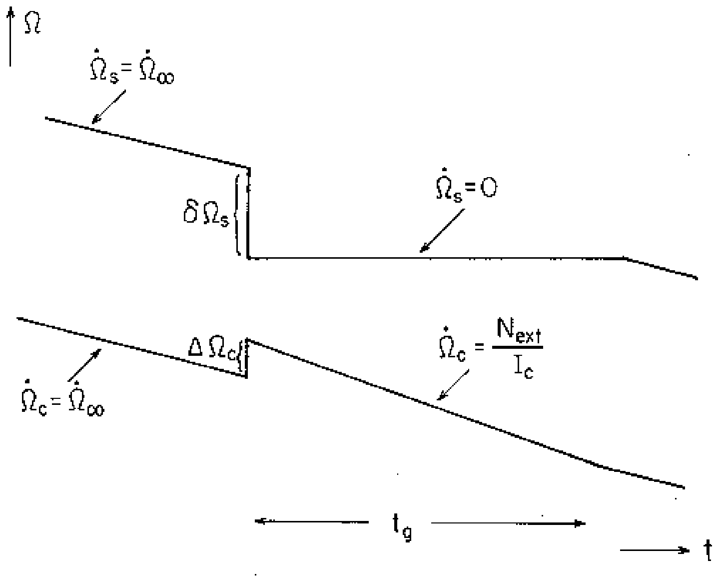

Figure 6.

The Vela pulsar's glitches and postglitch relaxation involves a superposition of vortex capacitor contributions, as in Fig.1, linear response with exponential relaxation, as in Fig.3, and the characteristic nonlinear response with $|\dot{\Omega}|_c \propto t$, as illustrated in Fig.6 (Alpar et al 1993). The latter term defines the interglitch timing behaviour extending to the next glitch: the healing of the lag to preglitch values signals return to the preglitch conditions. Hence the longest waiting time t_g should signal the arrival of the next glitch. This is indeed borne out by the evaluation of the constant second derivative together with the glitch parameters. For the Vela pulsar glitches this yields estimates of the time to the next glitch which agree with the observed interglitch values to 20% on the average. We follow the notation of earlier work and denote the moment of inertia of the vortex capacitors (Fig.1 behaviour) with I_B, and the total moment of inertia of the stacked nonlinear creep regions responsible for the constant $\ddot{\Omega}$ (Fig.6) behaviour with I_A. The equations relating these model parameters to the observed glitch parameters are (Alpar 1995):

$$\Delta\Omega = (I_A/2I + I_B/I)\delta\Omega \tag{18}$$

$$\Delta\dot{\Omega}/\dot{\Omega} = I_A/I \tag{19}$$

$$\ddot{\Omega} = (I_A/I)(\dot{\Omega}^2/\delta\Omega) \tag{20}$$

These equations yield

$$\ddot{\Omega} = (\beta + 1/2)((\Delta\dot{\Omega}/\dot{\Omega})^2{}_{-3}/(\Delta\Omega/\Omega)_{-6})(\dot{\Omega})^2/\Omega \tag{21}$$

$$n \equiv \Omega\ddot{\Omega}/(\dot{\Omega})^2 = (\beta + 1/2)((\Delta\dot{\Omega}/\dot{\Omega})^2{}_{-3}/(\Delta\Omega/\Omega)_{-6}) \tag{22}$$

Here $\beta = I_B/I_A$ and glitch parameters are scaled to characteristic values, e.g. the fractional change in rotation rate is $10^{-6}(\Delta\Omega/\Omega)_{-6}$. The time to the next glitch can be expressed as

$$t_g = \delta\Omega/|\dot{\Omega}| = 2 \times 10^{-3}(\beta + 1/2)\tau\,((\Delta\Omega/\Omega)_{-6}/(\Delta\dot{\Omega}/\dot{\Omega})_{-3}) \tag{23}$$

where $\tau = \Omega/2|\dot{\Omega}|$ is the pulsar's spindown age.

Pulsars other than the Vela pulsar also have large positive second derivatives. The statistics of large ($\Delta\Omega/\Omega > 10^{-7}$) glitches is consistent with the hypothesis that all pulsars older than Vela have similar glitch behaviour, with glitch intervals and relaxation times scaling with $|\dot{\Omega}|^{-1}$ (Alpar and Baykal 1994). The interglitch behaviour of older pulsars should then also be characterized by a constant second derivative of the rotation frequency after the postglitch transients are over. The vortex creep theory predicts that as a pulsar cools, nonlinear response will characterize more and more moment of inertia in the pinned superfluid, replacing linear response. Exponentially relaxing transients will be replaced with the characteristic response of nonlinear creep, which gives the slow recovery of the spindown rate at a constant second derivative of the rotation frequency. Even without such a transition to nonlinear response, if all relaxation and interglitch timescales are determined by neutron star physics except for their dependence on the external torque, they should scale with $|\dot{\Omega}|^{-1}$. In the Vela pulsar the relaxation times of postglitch transients range from 10 hours to 32 days while the time between glitches is of the order of two years. Throughout this interglitch interval the nonlinear response continues at constant second derivative. A typical pulsar will be glitching at intervals of several hundred years. Thus, scaling from the Vela pulsar, there is only a 5% chance that a random timing observation of the pulsar will fall in the epoch of transients following a glitch. Recent measurements of second derivatives corresponding to anomalously high braking indices, $n > 3$, can be understood as due to interglitch recovery of nonlinear creep. The agreement with the model developed for the detailed fits to the Vela pulsar's interglitch behaviour is remarkable.

Table 1 lists all glitches with $\Delta\Omega/\Omega > 10^{-7}$ with subsequent $\ddot{\Omega}$ measurements reported by Shemar and Lyne (1996). The observed value of $\ddot{\Omega}$ and the corresponding value of the braking index are listed together with the glitch parameters $\Delta\Omega/\Omega$ and $\Delta\dot{\Omega}/\dot{\Omega}$ and the value of β implied. We find that β is between 0.4 and 6 for all cases. This strongly supports the idea that Vela-like glitch behaviour is common to all pulsars older than Vela. This idea was first proposed for the Geminga pulsar, whose $\ddot{\Omega}$ was explained by assuming Geminga has Vela like glitches (Alpar, Ögelman and Shaham 1993). The limited variation in β may indicate that the ratio of capacitive to resistive parts of the vortex current network may vary between neutron stars, reflecting differences in the build-up history of the vortex capacitor network. Part of the variation may also be due to contamination by transients in the older pulsars.

A particularly interesting pulsar in Table 1 is PSR 1737-30, which has an age and glitch activity comparable to the Vela pulsar's, and has been observed to glitch repeatedly. Thus it has observed values of both $\ddot{\Omega}$ and the interglitch interval t_g. When the model is applied to the large glitches of PSR 1737-30, with $\Delta\Omega/\Omega > 10^{-7}$, one can use the β values obtained from the observed $\ddot{\Omega}$ to estimate the time between two consecutive glitches. These model estimates are compared with the observed interglitch intervals, showing an encouraging agreement to 20% for the two pairs of consecutive large glitches for which data are available.

There is statistical evidence that all pulsars older than the Vela pulsar have glitches of size similar to those of Vela, with glitch rates scaling as expected with the spindown rate. The only observed glitching pulsar younger than the Vela pulsar, the Crab pulsar, has quite a different glitch signature

TABLE 1.

Glitch	$(\Delta\Omega/\Omega)_{-6}$	$(\Delta\dot{\Omega}/\dot{\Omega})_{-3}$	$\ddot{\Omega}$	β	n	t_g
1727-33	3.07(1)	9.7(7)	628	0.74	38	253 yrs
47990(20)						
1737-30	0.427(10)	4.1(4)	1098	4.1	181	341 d
46991(40)						290±44 d
1737-30	0.6019(5)	2.3(2)	346	6.	57	604 d
47670(4)						516±10d
1800-21	4.073(16)	9.1(2)	867	0.4	18	17 yrs
48245(20)						
1823-13	2.718(26)	6.8(1)	1200	1.5	34	18 yrs
46507(60)						
Geminga	1.*	5.*	1-3	0.5	10-30	100-300 yrs

Glitches with $\Delta\Omega/\Omega > 10^{-7}$ and measured postglitch $\ddot{\Omega}$ values in units of 10^{-24} rad s^{-3}. The first column lists the pulsar names and glitch dates in MJD. Glitch parameters are taken from Shemar and Lyne (1996). n is the braking index. The model parameter β is defined after Eq.(22). t_g is the time to the next glitch estimated with Eq.(23). For PSR B1737-30, the observed time intervals to the next glitch are also listed, with errors due to the uncertainties in glitch dates. The last row applies the model to Geminga with assumed glitch parameters marked with asterisks.

and activity from Vela. The Crab pulsar's interglitch intervals do not agree with the vortex unpinning model. Vortex unpinning with vortex trap formation triggered by crust breaking (starquakes) can explain the Crab pulsar glitches (Alpar et al 1996). The comparison of Crab pulsar with Vela and other pulsars has led to an evolutionary picture of glitch behaviour (Alpar and Pines 1993, Alpar 1995). According to this picture the Crab pulsar is in the process of establishing a network of vortex capacitors. A pulsar at Vela's age already has a network that has saturated its pinned superfluid regions and controls a moment of inertia which is roughly that of all the pinned superfluid, a few percent of the total moment of inertia of the star. Such a saturated network can sustain collective unpinning events in which vortex unpinning builds into an avalanche in which all traps participate. Vela and older pulsars have large glitches with the occasional small glitch for network rearrangement. The Crab pulsar has small ($\Delta\Omega/\Omega \sim 10^{-9} - 10^{-8}$) glitches which are simultaneous crust breaking and vortex unpinning events. The breaking of the crust introduces new vortex traps. The persistent shift is just the moment of inertia of these trap regions as a fraction of the star's total moment of inertia. To build up to the total moment of inertia fraction of a few percent inferred from the Vela pulsar glitches will take of order 10^2 Crab like glitches, and this initial network building phase will last some thousands of years at an average interglitch time of order 10 years. The statistics of the small glitches from old pulsars is of great interest to test this evolutionary picture, which predicts that small glitches take place only occasionally in old pulsars with a rate determined by crust breaking as a trigger, while Vela type large glitches occur in all pulsars, no matter how old. An alternative model for glitches assigns the role of trigger to magnetic flux migrating from the core of the neutron star, and inducing stresses on the crust (Ruderman 1993, Ruderman, Zhu and Chen 1997). In this model the difference between the glitch signatures of the Crab and Vela pulsars, and the evolution of glitch behaviour is linked to the difference in the dynamics of the magnetic field. While not having a natural explanation for the constant second derivatives, this model is consistent with triggering vortex unpinning type glitches and the subsequent response of vortex creep. The distinction between the models lies in the predictions regarding the dynamics and glitch statistics of the older pulsars. In particular if magnetic field decay accompanies spindown, the long term braking index of pulsars should be close to 5 instead of 3. Ruderman's model also predicts that glitch activity stops in the oldest pulsars. It will be interesting to test these distinctions with the statistics of the pulsars' $P\dot{P}$ diagram and with pulsar glitch statistics.

4. Energy Dissipation in Glitches and Vortex Creep

The energy dissipation accompanying the angular momentum transfer in a glitch is

$$\Delta E_{diss} = \omega \Delta J = (I_A/2I + I_B/I)\omega\delta\Omega \tag{24}$$

where ΔJ is the angular momentum transfer across the preglitch lag ω between the pinned superfluid and the crust. This energy dissipation takes place in the pinned crust superfluid, at densities $\sim 10^{14}\,\mathrm{gm\,cm^{-3}}$. A thermal luminosity

$$\Delta L = \Delta E_{diss}/\tau_{th} \tag{25}$$

will emerge from the neutron star surface on the timescale τ_{th}, the thermal conduction time from the inner crust to the surface. With the Vela pulsar's glitch parameters, $\Delta E_{diss} \sim 10^{39}$ ergs, and with estimates of $\tau_{th} \sim 10^6$ s (Van Riper et al 1991) results of recent target of opportunity observations with ROSAT and XTE to look for excess thermal x-rays following the 11th glitch of the Vela pulsar will be of great interest.

The continuous rate of energy dissipation in vortex creep is:

$$\dot{E}_{diss} = I_A \omega |\dot{\Omega}| \tag{26}$$

This is significant only for nonlinear coupling between components of the star: in that case $\omega > \delta\Omega \sim 10^{-2}$ as determined from the Vela pulsar glitches, giving $\dot{E}_{diss} \sim 10^{31}$ ergs s^{-1} for this pulsar. Linear coupling would give $\omega = |\dot{\Omega}|\tau$ and $\dot{E}_{diss} = I_A(|\dot{\Omega}|)^2\tau \sim 10^{30}$ ergs s^{-1} even for a relaxation time $\tau = 1$ yr in the Vela pulsar, and would be much smaller in older pulsars. Taking into account nonlinear coupling, the contribution of $\dot{E}_{diss}$ to the X-ray luminosity is small compared to the cooling luminosity in young neutron stars, and somewhat smaller but comparable to the observed thermal luminosity (Ögelman 1995) in the Vela pulsar. For older pulsars $\dot{E}_{diss}$ will determine the neutron star's surface temperature (Alpar, Nandkumar and Pines 1985, Shibazaki and Lamb 1989, Ögelman 1995). This may provide a possible detection channel for the galactic population of "old" neutron stars. There is no evidence for decay of the dipole magnetic field in the population of observed ordinary (ie not millisecond) pulsars (eg Bhattacharya et al 1992, Hartman et al 1996). After the pulsar activity turns off as a result of spindown, the neutron star still experiences a dipole radiation torque. There will be energy dissipation due to the lag in rotation rate between the crust and the pinned superfluid. Noting that $\omega > \delta\Omega$, we find

$$\dot{E}_{diss} > (\Delta\Omega/\Omega)\, \dot{E}_{rot}/(\beta + 1/2) \sim 10^{-6} \dot{E}_{rot} \tag{27}$$

for the energy dissipation rate by vortex creep in an old neutron star which has occasional large glitches. $\dot{E}_{rot} = I\Omega|\dot{\Omega}|$ denotes the rotational energy loss rate. Using approximate expressions $\Omega \cong 6\, t_6{}^{-1/2}$rad s^{-1} and $\dot{\Omega}_{-13} \cong -t_6{}^{-3/2}$rad s^{-2} for a neutron star of moment of inertia $I = 10^{45}$ gm cm^2 and radius 10^6cm at age $t_6 \times 10^6$yrs subject to spindown by dipole radiation with a magnetic field $B = 10^{12}G$, the thermal luminosity $\dot{E}_{diss}$ and surface temperature T_s are:

$$\dot{E}_{rot} \cong 6 \times 10^{32} t_6{}^{-2} \mathrm{ergs\,s}^{-1} \tag{28}$$

$$\dot{E}_{diss} > 6 \times 10^{26} t_6{}^{-2} \mathrm{ergs\,s}^{-1} \tag{29}$$

$$T_s = (\dot{E}_{diss}/4\pi R^2 \sigma)^{1/4} > 3 \times 10^4 K\; t_6{}^{-1/2}. \tag{30}$$

There may be 10^5 neutron stars of age 10^7 years, luminosity $> 6 \times 10^{24}$ ergs s^{-1} and $T_s > 10^4 K$, and 10^6 neutron stars of age 10^8 years, luminosity $> 6 \times 10^{22}$ ergs s^{-1} and $T_s > 3 \times 10^3 K$, an interesting prospect of abundant but dim thermal point sources in the uv, optical and infrared bands.

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